# Math 3450 - Test 2

Name:________________________________________

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1. [16 points - 4 each] Fill in the rest of the definition.

(a) Let $A$ and $B$ be sets and $f : A \to B$. We say that $f$ is one-to-one if

(b) Let $A$ and $B$ be sets and $f : A \to B$. We say that $f$ is onto if

(c) Let $A$ and $B$ be sets and $f : A \to B$. Let $X \subseteq A$. We define the image of $X$ under $f$ to be

$f(X) =$

(d) Let $A$ and $B$ be sets and $f : A \to B$. Let $Y \subseteq B$. We define the inverse image of $Y$ under $f$ to be

$f^{-1}(Y) =$

2. [10 points - 5 each] Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^2 - 2$.

(a) Compute $f([0, 1])$.

(b) Compute $f^{-1}([-3, 0])$.

3. [20 points - 5 each] Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ and $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be given by the formulas $f(m, n) = (m + n, n^3)$ and $g(m, n) = (2m + 1, n)$.

(a) Compute $g(0, 1)$ and also compute $(g \circ f)(1, 1)$.

(b) Give a formula for $(g \circ f)(m, n)$.

(c) Prove that $g$ is one-to-one.

(d) Show that $g$ is not onto.

4. [10 points] Pick <u>ONE</u> of the following. If you do both then I will grade A.

A) Consider the function $\pi_4 : \mathbb{Z} \to \mathbb{Z}_4$ given by the formula $\pi_4(x) = \overline{x}$. Let $Y = \{\overline{2}\}$. Prove that $\pi_4^{-1}(Y) = \{4k + 2 \mid k \in \mathbb{Z}\}$.

B) Let $S = \mathbb{N} \times \mathbb{N}$. Define the relation $\sim$ on $S$ where $(a, b) \sim (c, d)$ if and only if $a + d = b + c$. You can assume that $\sim$ is an equivalence relation, no need to prove it. Define the operation $\overline{(a, b)} \oplus \overline{(c, d)} = \overline{(a + c, b + d)}$. Prove that $\oplus$ is well-defined on the set of equivalence classes.

5. [10 points] Pick ONE of the following. If you do both then I will grade A.

A) Let $A$ and $B$ be sets and $f : A \to B$. Prove that if $W \subseteq A$ and $Z \subseteq A$ then $f(W \cup Z) = f(W) \cup f(Z)$.

B) Let $A$, $B$, and $C$ be sets and $f : A \to B$ and $g : B \to C$. (i) Prove that if $f$ and $g$ are both onto, then $g \circ f$ is onto. (ii) Prove that if $f$ and $g$ are both one-to-one, then $g \circ f$ is one-to-one.